A Study of
Path Topology and its Analogues

A Synopsis

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By

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1 Introduction

Euclidean space is an appropriate mathematical model to study the Classical Mechanics in which time is taken as universal and does not depend on the state of motion of observer but this theory found to be inconsistent with the propagation of light as electromagnetic radiation. In 1905, Albert Einstein proposed the theory of Special Relativity which overcomes this drawback. Hermann Minkowski, a German mathematician, one of Albert Einstein's teachers, observed that the Special Theory of Relativity could be understood geometrically as a theory of the four dimensional spacetime (later known as Minkowski space). But this theory is applicable only when gravity of the observer is neglected thus Minkowski spacetime is generally regarded as the appropriate arena to formulate those laws of the physics that do not refer specifically to gravitational phenomena hence, Minkowski space is sometimes mentioned as a flat spacetime. To get a better understanding of the physical universe, Albert Einstein established theory of General Relativity which describes gravitation in terms of the curved four dimensional spacetime. The four dimensional Lorentz manifold is one of the most important models for this theory [18].

Quantum Field Theory is developed on Minkowski space, a flat spacetime then it is extended to the theory of curved spacetime by writing the theory in covariant form. Once the symmetries of Minkowski space are removed, some of the central ideas in the Quantum Field Theory are no longer be seemed so crucial; in particular, the notions of vacuum and particles lose their privileged positions [9].

String Theory is consistently defined in ten dimensions, six of which are curled up in some small internal compact manifold. The procedure of linking this manifold to four dimensional physics is called string compactification. The idea of compactication arises naturally to resolve the discrepancy between the critical ten dimensions and the number of observed four dimensions. Since M_{10} is dynamical, there can be solutions, consistent with the requirements imposed by local scale invariance on the world-sheet, which make the world appear four-dimensional. The simplest possibility is to have a background metric such that spacetime takes the product form $M_{10} = M_4 \times K_6$, where M_4 is four dimensional Minkowski space and K_6 is a compact space which is called internal manifold [8]. The simplicity and ele-

gancy of Minkowski space provides a unified and logically consistent picture of numerous physical phenomena in the domain of relativity, Quantum Field Theory and String Theory.

Zeeman suggested many physically relevant non-Euclidean topologies on Minkowski space [31]. These topologies are further studied by Nanda [24, 25, 26, 27], Agrawal et al. [1, 2, 3, 5] and others. Further, the generalization of Zeeman's topologies to the curved spacetime was initiated by Göbel [15] and this study has been carried forward by Domiaty [11, 12], Hawking et al. [19] etc. The present proposal is focused on a detailed topological study of the physically relevant topologies defined on a curved spacetime.

2 Notation and Preliminaries

All the notation and basic definitions which are used later are collected in this section. For terms not explained explicitly, we refer to [6, 7, 20, 28, 29]. The symbols R and N denote the set of real and natural numbers respectively.

2.1 Minkowski Space

An *n-dimensional Minkowski space* M for $n \in N$ and $n > 1$, is the n-dimensional real vector space R^n with *Lorentz inner product*, where Lorentz inner product is a bilinear form $g : R^n \times R^n \longrightarrow R$ satisfying the following properties (i) symmetric, i.e. $g(x, y) = g(y, x)$ for all $x, y \in R^n$ (ii) non-degenerate, i.e. if $g(x, y) = 0$ for all $y \in R^n$ then $x = 0$ and (iii) there exists a basis $\{e_0, e_1, ..., e_{n-1}\}$ of R^n such that $g(e_i, e_j) \equiv \eta_{ij}$ =1, for $i = j = 0$; -1, for $i = j = 1, ..., n-1$ and 0 otherwise. An element $x \in M$ is called an *event*. For an event, $x \equiv \sum_{i=0}^{n-1} x_i e_i$, the coordinate x_0 is called *time component* and the coordinates $x_1, ..., x_{n-1}$ are called *spatial component* of x relative to the basis $\{e_0, e_1, ..., e_{n-1}\}$. Lorentz inner product induces an indefinite characteristic quadratic form Q on M defined by $Q(x) = g(x, x)$. Thus, $Q(x) = x_o^2 - \sum_{i=1}^{n-1} x_i^2$. The group of linear operators T on M which leave the quadratic form Q invariant, i.e. $Q(x) = Q(T(x))$ for all $x \in M$, is called the *Lorentz group*.

An event $x \in M$ is called *timelike*, *lightlike* (*null*) or *spacelike* according as $Q(x)$ is positive, zero or negative. A straight line is called a *timelike,*

lightlike or spacelike according as it is parallel to a timelike, lightlike or a spacelike vector.

The sets $C^T(x) = \{\ y \in M : y = x \text{ or } Q(y-x) > 0\ \}$, $C^L(x) = \{\ y \in M : Q(y-x) = 0\ \}$ and $C^S(x) = \{\ y \in M : y = x \text{ or } Q(y-x) < 0\ \}$ are called the *time cone, light cone (or null cone), and space cone* respectively at the point $x \in M$. Further, the sets $C^+(x) \equiv \{y \in C^T(x) : y_0 > x_0\}$ and $C^-(x) \equiv \{y \in C^T(x) : y_0 < x_0\}$ are called the *future time cone* and *past time cone* at $x \in M$ respectively.

For $x, y \in M$, x *chronologically precedes* y, denoted by $x \ll y$, if $y \in C^+(x)$. Let J denote an open interval in R. A curve $\alpha : J \to M$ is *future timelike curve* (or *past timelike curve*) at $t_0 \in J$ if it is continuous with respect to the Euclidean topology and there exists an open connected set U in J containing t_0 such that for $t_1 < t_0 < t_2$, $\alpha(t_1) \ll \alpha(t_0) \ll \alpha(t_2)$ (or $\alpha(t_2) \ll \alpha(t_0) \ll \alpha(t_1)$) for every $t_1, t_2 \in U$. Further, α is called a *future timelike curve* (or *past timelike curve*) if it is future timelike curve (or past timelike curve) at every $t_0 \in J$. A curve is called a *timelike curve*, if it is either a future timelike curve or a past timelike curve [28].

2.1.1 Topologies on Minkowski Space

The *Euclidean topology* on Minkowski space M is the topology generated by the basis $B = \{N_\epsilon^E(x) : \epsilon > 0, x \in M\}$ where N_ϵ^E denotes the ϵ neighbourhood of x. The *fine topology* on Minkowski space is the finest topology which induces the 1-dimensional Euclidean topology on every timelike straight line and 3-dimensional Euclidean topology on every spacelike hyperplane [31]. The *time topology* is the finest topology that induces Euclidean topology on every timelike straight line [25]. The *space topology* is the finest topology that induces Euclidean topology on every spacelike hyperplane. The topology which is generated by the basis $\{N_\epsilon^E(x) \cap C^T(x) : \epsilon > 0, x \in M\}$ is called the *t-topology*. The topology which is generated by the basis $\{N_\epsilon^E(x) \cap C^S(x) : \epsilon > 0,\ x \in M\}$ is called the *s-topology* [25]. The topology which is generated by the basis $\{N_\epsilon^E(x) \cap (C^T(x) \cup C^S(x)) : \epsilon > 0, x \in M\}$ is called the *f-topology* [1]. The A-topology is the finest topology on Minkowski space which induces 1-dimensional Euclidean topology on every timelike and lightlike line and 3-dimensional Euclidean topology on every spacelike hyperplanes [27].

2.2 Manifolds

A topological space S is said to be *locally n-Euclidean*, if for each $x \in S$ there exists a neighborhood U which is homeomorphic to an open subset of R^n. A Hausdorff, second countable space S which is locally n-Euclidean is called *n-manifold*. A 1-manifold is often called a *curve* and a 2-manifold is called a *surface* [23]. Each pair (U, ϕ), where U is an open subset of S and ϕ is a homeomorphism from U to an open subset of R^n, is called a *coordinate neighborhood or chart*. If $U \cap V$ is nonempty, coordinate neighborhoods (U, ϕ) and (V, ψ) are called C^∞*-compatible* if the functions $\phi o \psi^{-1}$ and $\psi o \phi^{-1}$ are diffeomorphisms of the open subsets $\phi(U \cap V)$ and $\psi(U \cap V)$ of R^n respectively [6].

The collection of compatible charts which cover a manifold S is called an *atlas on S*. An atlas $\Re$ is called *complete atlas* if it is maximal, i.e. any coordinate neighbourhood (V, ψ) compatible with every $U_\alpha \in \Re$ is itself in $\Re$. The topology defined by the open sets consisting of unions of sets of the form U_α belonging to complete atlas is called *manifold topology* [7].

A C^∞ or *differentiable (or smooth) structure* on a topological manifold S is a family $\Re \equiv \{U_\alpha, V_\alpha\}$ of coordinate neighbourhood such that (i) the U_α cover M, (ii) for any α, β the neighbourhoods $(U_\alpha\ ,\ \phi_\alpha\ )$ and $(U_\beta,\ \phi_\beta)$ are C^∞-compatible and (iii) any coordinate neighbourhood $(V,\ \psi)$ compatible with every $U_\alpha\ \epsilon\ \Re$ is itself in $\Re$. A C^∞ *manifold* is a topological manifold together with a C^∞-differentiable structure.

The *tangent space* T_pS to S at p is the set of all functions $X_p : C^\infty(p) \to R$ such that for all $\alpha, \beta \in R$ and $f, g \in C^\infty(p)$ enjoying the properties (i) $(X_p + Y_p)f = X_pf + Y_pf$, (ii) $(\alpha X_p)f = \alpha(X_pf)$, (iii) $X_p(\alpha f + \beta g) = \alpha(X_pf) + \beta(X_pg)$ and (iv) $X_p(fg) = (X_pf)g(p) + f(p)(X_pg)$. A *tangent vector* to S at p is an element of T_pS. The *tangent bundle*, $T(S)$ on S is the set of all tangent vectors at all points of S. A *vector field* X on S is a function assigning to each point $p \in S$ a vector $X_p \in T_pS$ [6].

A parametrized curve $\alpha : (a, b) \to S$ is said to be a *geodesic* if its velocity vector is constant for each $t \in (a, b)$. An *Exponential map* is a function

$exp_p : T_p(S) \to S$ such that it takes a tangent vector X_p to that point on unique geodesic determined by X_p at which the parameter takes the value $+1$.

2.2.1 Lorentz manifold

A *Lorentz manifold* is a pair $L \equiv (S, g)$, where S is a connected, Hausdorff, C^∞, real, n-dimensional manifold without boundary and g is a C^∞ Lorentz metric on S.
Let $N(p)$ be the family of all open, convex, normal neighbourhood of $p \in L$, i.e. for every $U \in N(p)$ there exists a neighbourhood $\tilde{U}$ at the origin of $Tp(S)$ such that the diffeomorphic image $U \equiv exp_p(\tilde{U})$ is an open convex normal neighbourhood of p. Set $\tilde{B}_p(\epsilon) \equiv \{X \in T_pS : ||X|| < \epsilon\}$, where $||\ ||$ is a norm in T_pL (which is independent of p). Then, for every $U \in N(p)$ there exists an $\epsilon_u = \sup\{\epsilon > 0 : \tilde{B}_p(\epsilon) \subseteq \tilde{U}\ \} > 0$ such that $0 < \epsilon \leq \epsilon_u$. Now, set $B_u(p, \epsilon) \equiv exp_p\tilde{B}_p(\epsilon)$. The *space cone* at origin of T_pL is defined as $\tilde{R}_p \equiv \{X \in T_pS : X = 0 \text{ or } g(X, X) > 0\}$ and the space cone at p in U is defined as $S(p, U) \equiv exp_p(\tilde{U} \cap \tilde{R}_p)$. Also, the space cone at p in $B_u(p, \epsilon)$ is defined as $R_u(p, \epsilon) = exp_p(\tilde{B}_p(\epsilon) \cap \tilde{R}_p)$ [11].

2.2.2 Spacelike curve

Let J be an interval in R. Then a curve $\gamma : J \to L$ is called a *spacelike curve* in a Lorentz manifold L and abbreviated as $\gamma \in \sum$, if γ is continuous and for every $t \in J$ there exists an open and connected neighbourhood W of $t \in J$ and a $U \in N(\gamma(t))$, such that $\gamma(W) \subseteq S(\gamma(t), U)$. If $\gamma \in \sum$ and J is compact, then γ is called a spacelike path in (S, g).

2.2.3 Non-manifold topologies on Lorentz manifold

The *path topology* on a Lorentz manifold is defined as the finest topology which induces manifold topology on every timelike curve [19]. Naber [28] proved it to be a generalization of t-topology defined on Minkowski space.

The *Zeeman topology* on a Lorentz manifold is defined as the finest topology that induces manifold topology on every timelike geodesic and spacelike hypersurface. Göbel defined this topology as a generalization of the Zeeman's fine topology on Minkowski space to Lorentz manifolds [15].

The *space topology* on a Lorentz manifold is defined as the finest topology which induces manifold topology on every spacelike curve. This topology is strictly finer than the manifold topology [17]. Göbel defined it as a generalization of Zeeman's s-topology on Minkowski space to Lorentz manifolds [12].

3 Survey of the Relevant Literature

In this section, the study carried out so far, related to the proposed work, has been presented, as reviewed from the literature.

In 1964, Zeeman [30] represented causality on Minkowski space M by means of partial ordering and proved that the group of all automorphisms preserving this partial ordering is generated by the Lorentz group and dilatations.

At the first sight, it is customary to take the Euclidean topology on Minkowski space M but it is not an appropriate choice of topology due to the fact that it is locally homogeneous while every point of M associates a light cone which separates the space and time vectors. Also, it incorporates a very large group of homeomorphisms which has no physical significance. As, there can be a homeomorphism that takes timelike vector to spacelike vector with respect to this topology. This leads to define non-Euclidean topologies on Minkowski space.

Physically relevant non-Euclidean topologies on Minkowski space were first defined by Zeeman in 1967. Zeeman defined a topology as the finest topology that induces the 3-dimensional Euclidean topology on every spacelike hyperplane and 1-dimensional Euclidean topology on time axis and named it as the fine topology. It enjoys properties like Hausdorffness, connectedness and locally connectedness. However, it is not normal, not locally connected and not first countable. Its group of homeomorphisms is equal to the group generated by the Lorentz group together with dilatations and translations. Instead of these properties, the fine topology also has some other physical advantages: (i) the path of particle that preserves the partial order under the fine topology is like the path of freely moving particle with finite number of collisions and (ii) discreteness of induced topology on

a light ray intuitively means that the track of the photon is discontinuous which agrees with the physical phenomenon of discrete emission and absorption of photons [31]. In 2007, Dossena revisited it and obtained that the n-dimensional Minkowski space with the fine topology is Hausdorff, separable, not normal, not locally compact, not Lindelöf, and not first countable. Also, he proved two dimensional Minkowski space with the fine topology is path connected but not simply connected space and obtained a characterization of its compact sets. Further, He obtained that first homotopy group of two dimensional Minkowski space with the fine topology to be nontrivial.

Zeeman [31] also suggested some other physically significant non-Euclidean topologies on Minkowski space namely t-topology, s-topology, f-topology, space topology and time topology and conjectured that their homeomorphism group is the group generated by the Lorentz group, translations and dilatations. Nanda [24] proved this conjecture for space topology [24], t-topology [25] and s-topology [25]. He [24] also found that M is not normal, not locally compact and not second countable in the context of both space topology and s topology.

Further, Agrawal et al. (2012) [3] obtained a characterization of the compact sets of n-dimensional Minkowski space with s-topology and proved it to be not simply connectedness for 2-dimensional case using the notion of Zeno sequences which was initially introduced by Zeeman for fine topology [31]. Also, n-dimensional Minkowski space with s-topology was obtained to be a separable, first countable, path-connected, not regular, not metrizable, non second countable, not compact and not Lindelöf space. Quite recently, S. Erosy [13] studied the interior and closure of some important subsets of Minkowski space with respect to s-topology and Euclidean topology.

Continuing the topological study of n-dimensional Minkowski space with the t-topology, denoted by M^t, Agrawal et al.[2] characterized its compact sets and proved it to be a Hausdorff, path connected, separable, first countable but not second countable, not normal, not compact, not locally compact, not paracompact, not metrizable and not locally n-Euclidean space. Quite recently, Agrawal et al. (2015) obtained that invariance of domain theorem does not hold in Minkowski space with the t-topology unlike the Euclidean case. But its consequence, i.e. no nonempty open subset of the n-dimensional Minkowski space with t-topology is homeomorphic to an open subset of the

m-dimensional Minkowski space with t-topology for $n \neq m$. The technique to prove these results is entirely different from that of Euclidean case [5].

Further, Nanda et al. [26] defined a topology on Minkowski space as the topology generated by the positive cone at origin and its translates and named it as the the K-topology. It is also known as the order topology. Even though, the order topology is neither T_1 nor comparable with Euclidean topology, it incorporates a very pleasant property that Minkowski space with order topology is simply connected. Its homeomorphism group is the set of elements which are order automorphisms. This leads to the question that order topology is minimal in the sense that no strictly coarser topology on M can have this homeomorphism group. But Nanda et al. [26] found the answer in negative.

Continuing the topological study of Minkowski space, Agrawal et al. [1] studied the Minkowski space with the f-topology and obtained the characterization of its compact sets using the notion of Zeno sequances. It was obtained to be Hausdorff, path connected, separable, first countable, non-regular, non-second countable, non-paracompact, non-Lindelöf, non-normal, non-locally compact and non-metrizable space. For the two dimensional case, it was obtained to be a non-simply connected space.

Over the past few decades, there have been various attempts at defining the physically relevant topologies on spacetime manifolds. The most natural topology on manifolds is the manifold topology. But this topology is inappropriate as (i) it does not give the distinction between the spatial and temporal directions, (ii) its homeomorphism group is very vast and consist many undesirable maps (iii) the set of continuous path contains many unphysical paths that allows switching between temporal and spatial direction. This leads to define the non-manifold topologies on spacetime. Göbel, in 1976, defined general relativistic analogue of the fine topology (which he calls Zeeman topology) and obtained its group of homeomorphisms to be the same as the group of homothetic transformations on spacetime and found its world lines to be the broken geodesics [15].

In the same year, a seminar of Rüdiger Göbel at Cambridge stimulated Hawking, King, and McCarthy to introduced a physically relevant non-manifold path topology on strongly causal spacetime. It is found to be more

appealing and manageable topology than Zeeman topology. It has been found that it is first countable, separable, Hausdorff, path connected, locally path connected but not regular, not normal, not locally compact and not paracompact. Also, it determines the causal, differential and conformal structure of spacetime. Malament [22] proved that this result is true for every spacetime. Naber [28] found that path topology on four dimensional Minkowski space, a strongly causal spacetime, is same as the Zeeman´s t-topology. Low [21] proved that Minkowski space with t-topology is not simply connected and generalized this result to Lorentz manifolds with path topology.

The generalization of s-topology on Minkowski space to Lorentz manifolds (named as space topology) was carried out by Göbel (1976) [17]. He obtained that its homeomorphism group for time orientable strongly causal Lorentz manifold is the group of all conformal C^∞ diffeomorphisms and conjectured that this statement remains valid even if the space is not strongly causal. Domiaty [12] shown this to be true and further conjectured that a Lorentz manifold with the space topology possesses the same group of homeomorphisms even if the condition of time orientability is omitted. He found that a Lorentz manifold with space topology is 0-semimetrizable, semistratifiable and locally 0-semimetrizable. He also proved that Minkowski space is an example of 0-semimetric space. He remarked this space to be Hausdorff, first countable, separable, path connected, locally path connected but not regular, however the proofs were not given [12].

Göbel [16] defined infinite variety of non-manifold topologies on Lorentz manifolds and remarked that all these topologies are Hausdorff, locally and globally path connected and none of these topologies is regular, normal, locally compact and paracompact without given proofs. One of these topology is recognized as generalization of time topology on Minkowski space. He obtained Zeeman's conjecture in the context of time topology on Minkowski space as a consequence of the result obtained for its generalize case on Lorentz manifolds. An alternate proof is obtained by Agrawal et al. in [4] using a simpler approach similar to one used in [28] for the computation of homeomorphism group of Minkowski space with path topology.

Fullwood [14] worked out that the need of manifold topology to define continuous timelike curves (used to define path topology) restricts the class of spacetime on which the path topology can be defined. He suggested a

new topology on spacetime by combining the idea of Hawking's path topology and Alexandrov topology, which is completely defined in terms of causal structure and also proved that distinguishing condition holds on a manifold if and only if this new topology is equivalent to path topology. Also found that homeomorphism group of this new topology on the spacetimes having chronology condition is the group of chronological and antichronological bijections and proved that this new topology determines the causal, differential and conformal structure for the spacetimes having distinguishing condition.

4 Motivation

While introducing physically relevant non-Euclidean topologies on Minkowski space, Zeeman [31] defined a topology on Minkowski space as the topology generated by the basis $\{N_\epsilon^E(x) \cap C^T(x) : \epsilon > 0, x \in M\}$ which was later named by Nanda [25] as the t-topology who found that its homeomorphism group is the group generated by Lorentz group together with translations and dilatations as conjectured by Zeeman. Later on, Hawking et al. [19] defined path topology on Lorentz manifolds as the finest topology which induces Euclidean topology on every timelike curves. Naber in [28] found that Hawking's path topology on Minkowski space is the same as t-topology. This leads to two formulations of t-topology on Minkowski space: one is in the terms of basic open sets as suggested by Zeeman and the other is in terms of the timelike curves as a special case of path topology. Naber [28] studied its homeomorphism group using the second approach and found it to be the Lorentz group together with translations and dilatations. Recently, Agrawal et al. [5] made use of this approach to solve the classification problem for Minkowski space with t-topology, i.e. no open set of n-dimensional Minkowski space with t-topology is homeomorphic to an open set of m-dimensional Minkowski space with t-topology.

The s-topology on Minkowski space which is defined as the topology generated by the basis $\{N_\epsilon^E(x) \cap C^S(x) : \epsilon > 0, x \in M\}$ has its homeomorphism group to be the group generated by Lorentz group together with translations and dilatations [25]. Further, the space topology on Lorentz manifolds is defined as the finest topology which induces Euclidean topology on every spacelike curves and is mentioned to be a generalization of s-topology on Minkowski space without obtaining spacelike curve formulation for s-

topology [12, 17]. Thus following the time like curve approach of Naber [28] for t-topology, an alternate formulation for s-topology in terms of spacelike curves can be worked out which could be helpful in the study of classification problem in the context of s-topology.

While studying physically relevant non-manifold topologies on Lorentz manifolds, Hawking et al. [19] and Göbel [15, 17] defined path topology [19], Zeeman topology [15], space topology [17] etc. on a Lorentz manifold. Göbel [16] introduced an infinite series of topologies on curved spacetime. One of the topologies of this series is recognized as a generalization of time topology defined on Minkowski space to Lorentz manifold. Topological properties of Lorentz manifold with path topology have been studied by Hawking et al. [19]. However, the topological properties of other generalizations have not yet been studied and we propose to undertake this study in the present proposal.

A further advancement of the preceding study is motivated by the fact that two different dimensional Minkowski spaces with t-topology are not homeomorphic [5], as this fact leads to a new type of manifold to be defined as a topological space which is locally homeomorphic to M_t, where M_t is n-dimensional Minkowski space with t-topology. We will call it as the t-manifold of dimension n. The study of these manifolds may lead to an alternate method for the study of path topology on Lorentz manifolds, besides providing a completely new object to study, important from physical as well as mathematical view point.

5 Proposed Direction of Work

The present proposal is focussed on the study of physically significant non-Euclidean topologies on Minkowski space, introduced by Zeeman [31] and their generalizations on Lorentz manifolds such as the path topology introduced by Hawking et al. [19] and Zeeman topology introduced by Göbel [15] among many others. To be precise, the proposed work is aimed at the study of the following:

1. Homeomorphisms between Minkowski spaces of same dimension with s-topology, obtaining "spacelike curve" characterization of s-topology.

2. Relationship between different dimensional Minkowski spaces with s-topology.

3. Invariance of domain theorem in Minkowski spaces with s-topology.

4. Topological properties - separation axioms, countability axioms, connectedness axioms etc. for the non-manifold space topology, Zeeman topology etc. on Lorentz manifolds.

5. t-manifold: it is proposed to be defined as a topological space which is locally homeomorphic to M_t, where M_t is n-dimensional Minkowski space with t-topology. In view of the fact that no two open sets of different dimensional Minkowski spaces with t-topology are homeomorphic, this definition makes sense.

6. Connection between t-manifold and Lorentz manifold with path topology.

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